Application to the Motion and Equilibrium of the Planar Kinematic Chains with Rotational Links with Clearances

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Abstract: Based on the algorithm presented in our previous papers [1], we now present their applications to a few problems.

1. Application

We consider the planar kinematical chain from the Figure 1

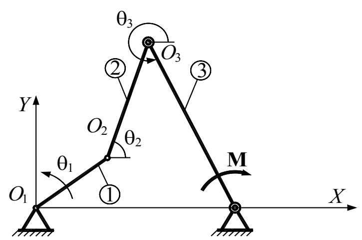

Figure 1. Quadrilateral mechanism with clearances in the kinematical joints O_3, O_4.

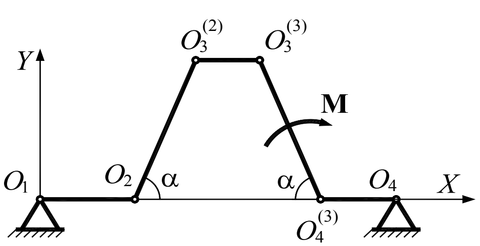

Figure 2.. Initial position of the quadrilateral mechanism with clearances.

Determine the equations of motion and the reactions for the mechanism $O_1O_2O_3O_4$ with clearances in the rotational kinematical joints O_3, O_4, Figure 1, knowing that the element O_1O_2 rotates with constant angular speed ω and the element O_3O_4 is acted by a torque with the moment M. We consider that the elements of the mechanism are homogeneous bar of constant cross section and we consider known the following parameters:

– dimensions $l_1 = O_1O_2$, $l_2 = O_2O_3$, $l_3 = O_3O_4$, $l_1 = l_2$;

– the coordinates X_{O_4}, $Y_{O_4} = 0$ of the articulation O_4;

– the clearances r_3, r_4 at the joints O_3, O_4;

– the masses m_3, m_4 and the moments of inertia J_2, J_3;

– the initial conditions, Figure. 2, at $t = 0$: $X_{O_4} = l_1 + 2l_2 \cos\alpha + r_3 + r_4$, $\theta_2 = \alpha$, $X_2 = l_1 + \frac{l_2}{2}\cos\alpha$, $Y_2 = \frac{l_2}{2}\sin\alpha$, $\theta_3 = -\alpha$, $X_3 = X_{O_4} - r_4 - \frac{l_2}{2}\cos\alpha$,

$\dot{X}_2 = 0$, $\dot{Y}_2 = l_1\omega$, $\dot{\theta}_2 = 0$, $\dot{X}_3 = \dot{Y}_3 = 0$, $\dot{\theta}_3 = 0$.

Numerical application for $l_1 = 0.2\,\mathrm{m}$, $l_2 = l_3 = 0.6\,\mathrm{m}$, $r_3 = r_4 = 0.005\,\mathrm{m}$, $m_2 = m_3 = 2\,\mathrm{kg}$, $J_2 = J_3 = 0.06\,\mathrm{kgm^2}$, $M = 40\,\mathrm{Nm}$, $\omega = 10\,\mathrm{rad/s}$.

Solution: Based on the previous relations of calculation, we can write in order the equalities $\tilde{x}_2^{(2)} = -\frac{l_2}{2}$, $\tilde{y}_2^{(2)} = 0$, $\tilde{U}_{2X} = \tilde{x}_2^{(2)}\cos\theta_2$, $\tilde{U}_{2Y} = \tilde{x}_2^{(2)}\sin\theta_2$,

$$\left[\tilde{\mathbf{B}}_2^{(2)}\right] = \begin{bmatrix} 1 & 0 & -\tilde{U}_{2Y} \\ 0 & 1 & \tilde{U}_{2X} \end{bmatrix}, \quad x_3^{(2)} = \frac{l_2}{2},$$

$y_3^{(2)} = 0$, $U_{3X}^{(2)} = x_3^{(2)}\cos\theta_2$, $U_{3Y}^{(2)} = x_3^{(2)}\sin\theta_2$, $x_3^{(3)} = -\frac{l_2}{2}$, $y_3^{(3)} = 0$, $U_{3X}^{(3)} = x_3^{(3)}\cos\theta_3$, $U_{3Y}^{(3)} = x_3^{(3)}\sin\theta_3$,

$$D_{3X} = \frac{X_3 + U_{3X}^{(3)} - X_2 - U_{3X}^{(2)}}{r_3},$$

$$D_{3Y} = \frac{Y_3 + U_{3Y}^{(3)} - Y_2 - U_{3Y}^{(2)}}{r_3},$$

$$\{\mathbf{D}_3\} = \begin{bmatrix} D_{3X} \\ D_{3Y} \end{bmatrix}, \quad \left[\mathbf{B}_3^{(2)}\right] = \begin{bmatrix} 1 & 0 & -U_{3Y}^{(2)} \\ 0 & 1 & U_{3X}^{(2)} \end{bmatrix},$$

$$\left[\mathbf{B}_3^{(3)}\right] = \begin{bmatrix} 1 & 0 & -U_{3Y}^{(3)} \\ 0 & 1 & U_{3X}^{(3)} \end{bmatrix},$$

$$\left[\mathbf{E}_3^{(2)}\right] = \{\mathbf{D}_3\}^T\left[\mathbf{B}_3^{(2)}\right], \quad \left[\mathbf{E}_3^{(3)}\right] = \{\mathbf{D}_3\}^T\left[\mathbf{B}_3^{(3)}\right],$$

$x_4^{(3)} = \frac{l_3}{2}$, $y_4^{(3)} = 0$, $U_{4X}^{(3)} = x_4^{(3)}\cos\theta_3$, $U_{4Y}^{(3)} = x_4^{(3)}\sin\theta_3$, $x_4^{(3)} = \frac{l_3}{2}$, $y_4^{(3)} = 0$, $U_{4X}^{(3)} = x_4^{(3)}\cos\theta_3$, $U_{4Y}^{(3)} = x_4^{(3)}\sin\theta_3$,

$$\left[\mathbf{B}_4^{(3)}\right] = \begin{bmatrix} 1 & 0 & -U_{4Y}^{(3)} \\ 0 & 1 & U_{4X}^{(3)} \end{bmatrix},$$

$$D_{4X} = \frac{X_{O_4} - X_3 - U_{4X}^{(3)}}{r_4},$$

$$D_{4Y} = \frac{Y_{O_4} - Y_3 - U_{4Y}^{(3)}}{r_4}, \quad \{\mathbf{D}_4\} = \begin{bmatrix} D_{4X} \\ D_{4Y} \end{bmatrix},$$

$$\left[\mathbf{E}_4^{(3)}\right] = \{\mathbf{D}_4\}^T\left[\mathbf{B}_4^{(3)}\right],$$

$$[\mathbf{B}] = \begin{bmatrix} \left[\mathbf{B}_2^{(2)}\right] & [\mathbf{0}] \\ -\left[\mathbf{E}_3^{(2)}\right] & \left[\mathbf{E}_3^{(3)}\right] \\ [\mathbf{0}] & -\left[\mathbf{E}_4^{(3)}\right] \end{bmatrix},$$

$$\{\mathbf{C}\} = l_1\omega\left[-\sin\omega t \;\; \cos\omega t \;\; 0 \;\; 0\right]^T,$$

$$\{\mathbf{q}\} = \left[X_2 \;\; Y_2 \;\; \theta_2 \;\; X_3 \;\; Y_3 \;\; \theta_3\right]^T,$$

$$\{\mathbf{R}\} = \left[H_2 \;\; V_2 \;\; N_3 \;\; N_4\right]^T,$$

$$[\mathbf{M}] = \begin{bmatrix} m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_3 \end{bmatrix}$$

and, since the matrix $[\mathbf{M}]$ is invertible, it results that the equation $[\mathbf{M}]\{\ddot{\mathbf{q}}\} = \{\mathbf{F}\} - [\mathbf{B}]^T\{\mathbf{R}\}$ separates into the equalities

$$\{\mathbf{R}\} = \left[[\mathbf{B}][\mathbf{M}]^{-1}[\mathbf{B}]^T\right]^{-1}\left\{[\mathbf{B}][\mathbf{M}]^{-1}\{\mathbf{F}\} - \{\dot{\mathbf{C}}\} + [\dot{\mathbf{B}}]\{\dot{\mathbf{q}}\}\right\}$$

$$\{\ddot{\mathbf{q}}\} = [\mathbf{M}]^{-1}\left\{\{\mathbf{F}\} - [\mathbf{B}]^T\{\mathbf{R}\}\right\}.$$

Further on, with the aid of the notations $\{\mathbf{f}\} = [f_1 \;\; f_2 \;\; \ldots \;\; f_6]^T = [\mathbf{M}]^{-1}\left\{\{\mathbf{F}\} - [\mathbf{B}]^T\{\mathbf{R}\}\right\}$, $p_i = \begin{cases} q_i, \text{if } 1 \le i \le 6 \\ \dot{q}_i, \text{if } 7 \le i \le 12 \end{cases}$, one obtains from (1) the systems of first order differential equations $\frac{\mathrm{d}p_i}{\mathrm{d}t} = \begin{cases} q_i, \text{if } 1 \le i \le 6 \\ \dot{q}_i, \text{if } 7 \le i \le 12 \end{cases}$, which, by numerical integration using the fourth order Runge–Kutta, leads us to the results captured in Figures. 3–12

From the Figure 3 we observe that the variations of the kinematical parameters O_2, O_3 in the case with clearance are small relative to the case without clearance. These variations diminish when the clearances r_3, r_4 diminish.

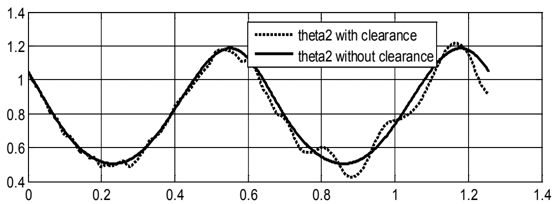

Figure 3. Time history of the parameters q_2 in the cases without and with clearances.

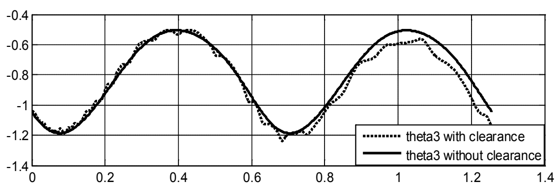

Figure 4. Time history of the parameters q_3 in the cases without and with clearances.

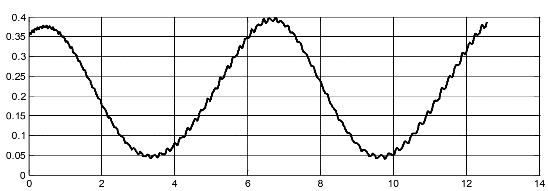

Figure5. Time history of the parameter X_2 .

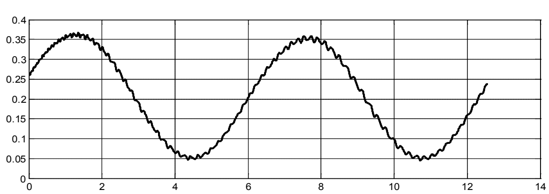

Figure 6. Time history of the parameter Y_2 .

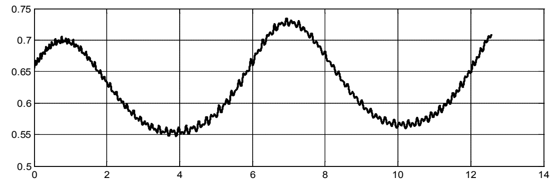

Figure 7. Time history of the parameter X_3 .

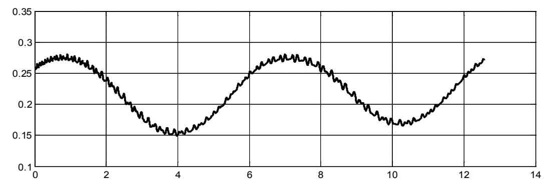

Figure 8. Time history of the parameter Y_3 .

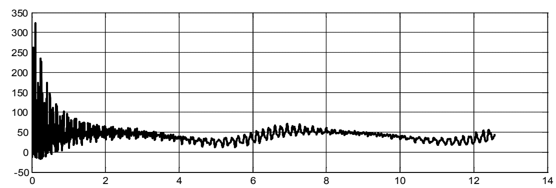

Figure 9. Time history of the reaction H_2 .

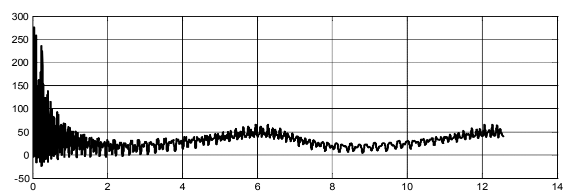

Figure 10. Time history of the reaction V_2 .

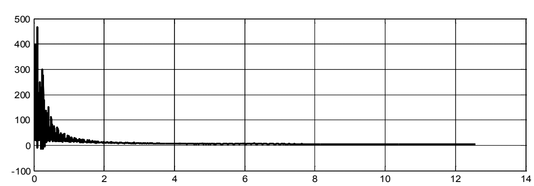

Figure 11. Time history of the reaction N_3 .

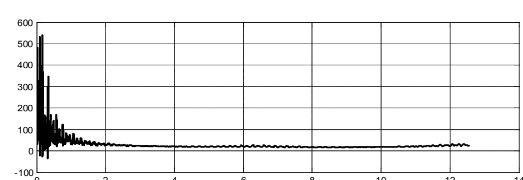

Figure 12. Time variation of the reaction N_4 .

2. Conclusions

In this paper we presented one application concerning the motion and the equilibrium of the planar chains with rotational linkages with clearances. The applications are complete and numerically solved.

The numerical applications solved here confirm the statements mentioned on the algorithm presented in our previous papers.

Acknowledgement

This work was supported by Romanian Ministry of Education and by Executive Agency for Higher Education, Research, Development and Innovation Funding, under projects PCCA 2-nr. 166/2012 and ENIAC 04/2014.

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